

# Teaching Quadratic Functions in Classroom and Online Using Mathematical Software Tools

Valentina Kostić<sup>1\*</sup>, Tanja Sekulić<sup>2</sup>

<sup>1</sup> Academy of Applied Technical and Preschool Studies Nis - Department Pirot, Pirot, Serbia

<sup>2</sup> Technical College of Applied Sciences in Zrenjanin, Zrenjanin, Serbia

\* [22mathgim@gmail.com](mailto:22mathgim@gmail.com)

**Abstract:** *In this paper, we present our experience and the materials used for teaching quadratic functions in the classroom and online. For the purpose of the research, we developed dynamical teaching materials prepared using mathematical software GeoGebra. The GeoGebra material for the quadratic function is discussed and described in detail. The experiences of the students and teachers concerning the used GeoGebra material are also presented in the paper. The results and impressions during the application of this material, in the classroom and online, were positive with the implications for further improvement and future applications.*

**Keywords:** *quadratic function; GeoGebra; dynamical materials*

## 1. INTRODUCTION

Digital technologies have a significant impact on education, educational content, methods, and techniques, curricula, and programs, as well as on the entire organization of teaching and learning. In modern education, the question is no longer whether digital technologies should be applied, but the main task is to find optimal solutions for how and in what way to apply new technologies in teaching practice, in order to improve the quality of teaching and make learning more efficient.

Taking into account the new situation related to the pandemics, new conditions and restrictions have been imposed regarding the realization of the teaching process. Digital technologies have gained even more importance and their application has significantly contributed to the quality of teaching and learning in the conditions of hybrid and online education [1].

Teaching materials should be adapted to new conditions and types of teaching. On the example of the realization of the theme: Quadratic function, we have shown one way of integrating digital technologies into teaching. For the needs of teaching and learning, we have developed dynamical teaching material using GeoGebra mathematical software and described how it can be applied in different teaching conditions.

The impressions and reactions of students and teachers to the proposed GeoGebra material were discussed and further guidelines were given for the improvement and application of this type of material in teaching and learning.

## 2. GEOGEBRA SOFTWARE

GeoGebra is a dynamical mathematical software that combines elements of geometry, algebra, analysis, statistics, probability, and spreadsheets, so it can be applied to study a wide range of different mathematical topics. GeoGebra has exceptional educational potential and can be implemented at all levels of education, from preschool to university.

The following features of this software are especially important for teaching and learning mathematics: the use of multiple representations, visualization, dynamics, and interactivity. In the GeoGebra environment, mathematical objects are created using appropriate tools and are displayed in multiple representations: graphical, algebraic, and numerical. All representations of an object are dynamically linked and automatically change when any one of them changes. In addition, objects can be animated into a moving image - a dynamical visual representation. GeoGebra is a powerful tool for dynamical visualization and working with multiple representations and has the potential to actualize visual thinking and facilitate the connection of visual knowledge with the formal-symbolic language of mathematics [2].

The visual-dynamical and interactive functionality of GeoGebra software enables teachers to create a stimulating environment for studying functions [3]. GeoGebra is a cognitive tool that can take over the execution of procedural activities in less time, thus leaving students more time to make connections between different functions' representations, think critically, and build a better understanding of

concepts and ideas. This educational software has the potential to encourage active student participation through visual-manipulative activities, experimentation, interactive exploration, discussions, and collaborative learning

### 3. TEACHING QUADRATIC FUNCTIONS

The quadratic function is one of the required subjects studied in high school. It is of great importance for students to fully master the quadratic function, which means understanding its features and possible applications.

In our high schools, the teaching practice is to explain to the students the definition of quadratic function, give different examples, and show its different forms. The most common form used is:

$$y = ax^2 + bx + c \quad a \neq 0. \quad (1)$$

Then, we explain its features like its roots, the basic shape (parabola), vertex (minimum, maximum value), sign, monotonicity, and the shape concerning the cases when the parabola opens upward or downward.

To illustrate all these features to students we usually draw all the cases together, like it is shown in Fig. 1.

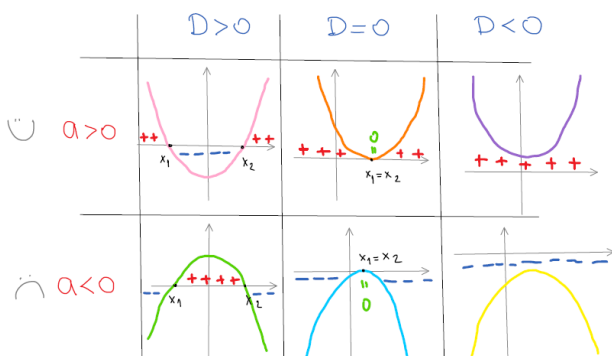


Figure 1. Different cases of the quadratic function

One of the most important requirements we put before students is that they properly understand the connection and dependency of coefficients  $a, b$ , and  $c$  from the standard form (1). For that purpose, we encourage our students to practice by exploring various cases of quadratic functions and sketching their graphs.

Mostly, they were given tasks to sketch two different cases of a quadratic function on one paper, usually with the same roots but with different shapes (Fig. 2).

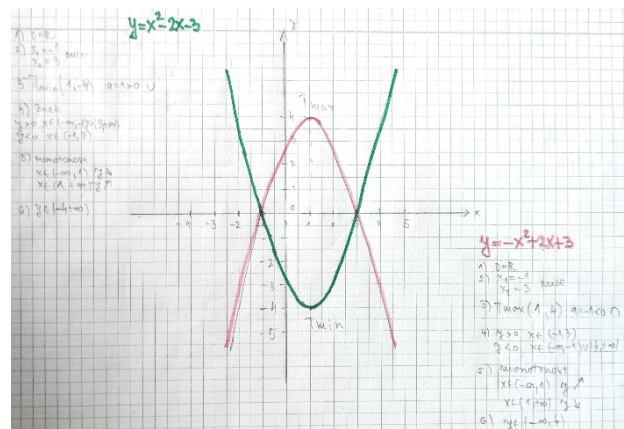


Figure 2. Students' sketch of the quadratic functions

In this way, the students can observe how changing the value of the coefficients  $a, b$ , and  $c$  affects the change in the shape of the graph and the properties of the quadratic function.

The students are happy to solve tasks of this type, and when they practice examining and sketching function graphs in this way, they can later easily apply the acquired knowledge.

However, we always looked for more efficient methods concerning teaching and learning quadratic functions where the students could be more actively involved in the process of learning and where they could explore quadratic functions by manipulating the coefficients  $a, b$ , and  $c$ , and observe the changes on the function graph at the same time.

Lately, especially considering teaching and learning in pandemic conditions, the search for more efficient methods became even more emphasized because we had to take into account not only learning in the classroom but also teaching and learning in online conditions.

### 4. METHOD

In order to improve the teaching of mathematics in high school, and taking into account the possible and available ways of teaching in the conditions of the pandemic, we tried to integrate mathematical software into the teaching to create additional learning materials.

#### 4.1. GeoGebra-assisted quadratic function learning

The idea was to design dynamic materials using GeoGebra software because such materials give the possibility of changing parameters and manipulating them, and all of that can be displayed and experimented with in different cases. We already had positive experiences with GeoGebra dynamical materials in teaching mathematics and sciences online and in the classroom [1], [4], [5].

For the purpose of teaching and learning quadratic functions, we used the GeoGebra official website, where we have placed the dynamic quadratic function study material that we have created. Geogebra material is available for everyone to use and can be found at the following link:

<https://www.geogebra.org/m/zjsq8tes>

On the GeoGebra official site, our material for learning quadratic function is marked as the "activity", as shown in Fig. 3.



Figure 3. Activity for learning quadratic function on the official GeoGebra website

By clicking on the link for the activity, opens the dynamical material, shown in Fig. 4.

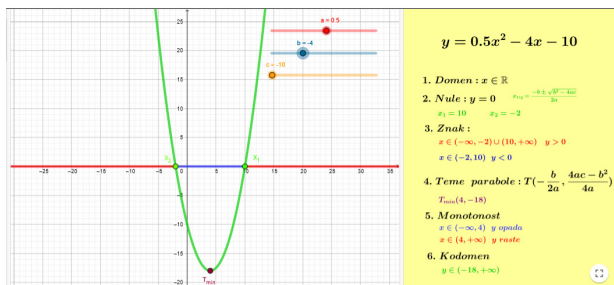


Figure 4. Dynamical material for learning quadratic function

It can be observed that this material consists of two windows. In the window on the left, there is a graph of the quadratic function, and in the upper right corner three sliders.

The sliders are the special feature of GeoGebra, constructed in order to enable the change of parameters values and by that gives the possibility to create dynamical materials.

In the case of the quadratic function material, we inserted three sliders, each one for the three coefficients  $a, b,$  and  $c$ . By dragging the point on the slider, the value of the coefficient connected with this slider is changing (Fig. 5).

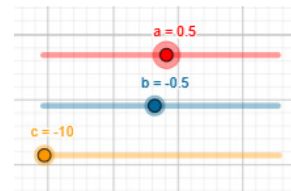


Figure 5. Sliders for the quadratic function coefficients

The window on the right contains all the features which correspond to the function from the left window like the expression of the quadratic function, roots, signs, vertex, monotonicity, etc. Fig. 6 shows the right window:

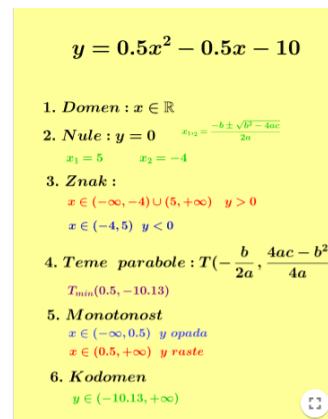


Figure 6. The right window appearance

It is important to emphasize that both windows are connected, meaning, that the change of parameters' values in the left window changes the functions' graph in the same left window, and simultaneously all the features of the function in the right window are changed.

In that way, many different cases of the quadratic function can be easily obtained and represented.

**Case 1 ( $a > 0, b > 0, c < 0$ )**

Case 1 is the standard example of the quadratic function. This is the case that illustrates the quadratic function with the two roots, vertex in minimum, its signs, and parabola opened upward. The students can also observe the values of the coefficients (Fig. 7).

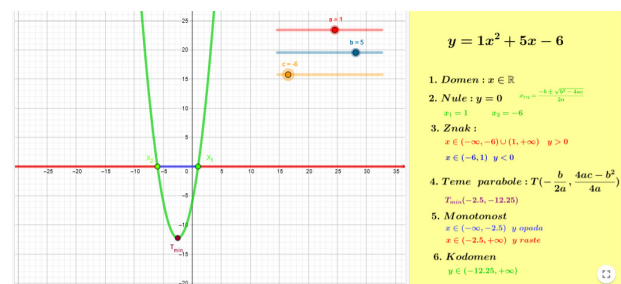
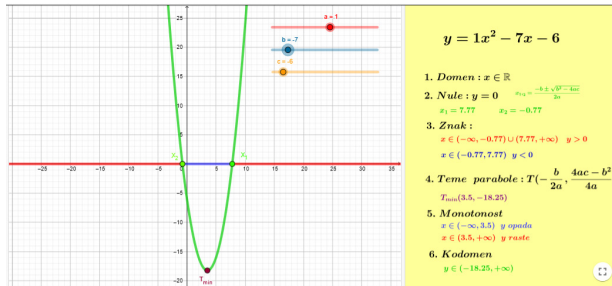


Figure 7. Case 1 ( $a > 0, b > 0, c < 0$ )

**Case 2 ( $a > 0, b < 0, c < 0$ )**

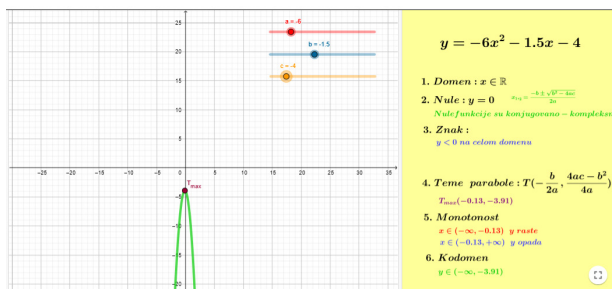
In Case 2 only one coefficient changed its value. The coefficients  $a > 0$  and  $c < 0$  did not change their signs, but the coefficient by  $x$  power one is now negative,  $b < 0$  and the students can observe how the functions' graph and its features changed compared to Case 1 (Fig. 8).



**Figure 8.** Case 2 ( $a > 0, b < 0, c < 0$ )

**Case 3 ( $a < 0, b < 0, c < 0$ )**

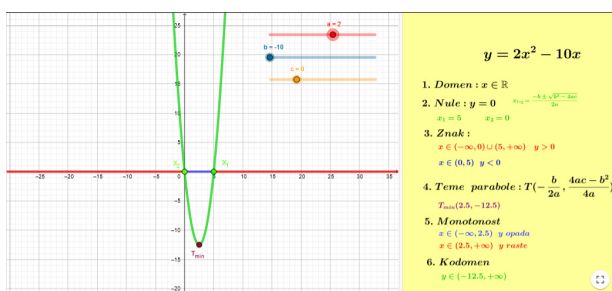
Case 3 illustrates what happens with the quadratic functions' graph when all the coefficients are negative (Fig. 9).



**Figure 9.** Case 3 ( $a < 0, b < 0, c < 0$ )

**Case 4 ( $a > 0, b < 0, c = 0$ )**

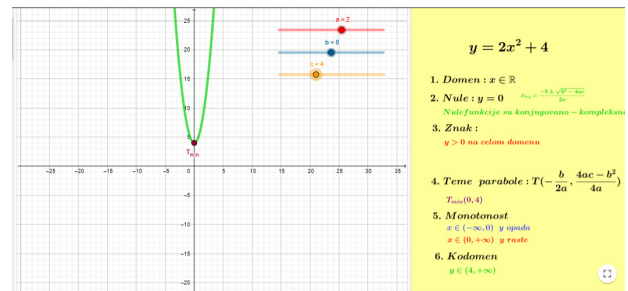
Case 4 illustrates the special type of quadratic function (whose roots are solutions of incomplete quadratic equations) where  $c = 0$  (Fig. 10).



**Figure 10.** Case 4 ( $a > 0, b < 0, c = 0$ )

**Case 5 ( $a > 0, b = 0, c > 0$ )**

Case 5 illustrates the other special type of quadratic function (whose roots are solutions of incomplete quadratic equations) where  $b = 0$  (Fig. 11).



**Figure 11.** Case 5 ( $a > 0, b = 0, c > 0$ )

Of course, in addition to these cases, there are many others that can be generated in our GeoGebra material. The point is, that the material that we proposed can help students to experiment with the graph of a quadratic function in a simple way, only by moving the sliders for coefficients from the function. In that manner, the students can obtain the solutions and conclusions related to the quadratic function.

**5. FINDINGS AND DISCUSSION**

The presented GeoGebra material was primarily used for the purpose of online teaching, during pandemics. The material was developed for the personal use of the teacher, as additional material for the high school online mathematics course. At first, the material was sent to students only by email, but concerning teachers' positive impressions of using this material, it was decided to make it accessible to all teachers and students, and consequently, it was placed on the official GeoGebra site. It was noticed, that the material was used by many other mathematics teachers who also said that the material significantly helped them in the teaching process, online and in the classroom.

The students also had very positive impressions of this material. They especially emphasize the possibility of experimenting with the different values of coefficients and the instant sketching of the function graph and display of functions' features. They also commented that the possibility to access the material at any time helped them a lot in the organizing and planning of the learning process.

The observations of teachers were also in favor of using this material. First, they commented that the teaching process was more effective and the students were very active and more interested in the topic they learned when using GeoGebra material. Also, the teachers detected that the attitudes of the students toward homework and other learning responsibilities were better compared to the ones before. Also, the teachers noticed that the grades of the students remained at the same level, meaning that the GeoGebra dynamical material helped them to overcome the

obstacles that may have appeared during the online teaching and learning process.

Although this material is used in online teaching and learning during the pandemic, when we returned to the classrooms we also continued to use the same material as support for the regular teaching process as a response to our students' requests and to our positive experiences.

## 6. CONCLUSION

Finding the most appropriate way for teaching mathematics in high school can be very challenging, especially when the conditions are more than unusual, as we witnessed during the last two years of the pandemic.

It was of great importance to find the appropriate way, for the teaching process and for learning, so that the achievements of the students remain at a satisfactory level.

Contemporary software tools have proven to be of great help to teachers and students also. In teaching mathematics, specialized mathematical software, such as GeoGebra are particularly useful for teaching and learning mathematics because of its dynamic nature and the possibility for multiple representations.

We have presented and described the GeoGebra material that we have created and used for teaching quadratic function in high school. The experiences using GeoGebra materials were positive, concerning both, teachers and students. The students said that the GeoGebra material we have made for quadratic function helped them to better understand it, but most important of all, they emphasized that the opportunity to experiment on their own using this dynamical material helped them the most.

The students' and our positive experiences from the teaching and learning process using GeoGebra materials as support in the classroom and online, lead us to the conclusion that the implementation

of mathematical software and dynamical materials can contribute in many ways to teaching and learning process.

In the future, we will continue with the implementation, improvement, and development of the digital materials and the software we use. Also, we will share our good practice with colleagues through the exchange of experiences and materials we are yet to develop.

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