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Application of statistics for the analysis of results achieved in primary education

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Abstract: - *Statistics is one of the most applicable mathematical disciplines dealing with data collecting and processing as well as the analysis of the achieved results. Statistical packages enable quick processing of these data and performing statistical analyses. In this paper, the application of Statistica software package enabled processing of the data that included 80 pupils. The achievements of pupils from the subject of technical and informatics education were examined by tests. Descriptive statistical procedures were used in data processing. Appropriate statistics were used to carry out the analysis and adequate interpretations of the obtained results were provided. The test results indicate a normal distribution of data as well as gender independence relative to the area being tested by testing.*

Keywords: *statistics, descriptive statistics, hypothesis of testing, statistical analysis.*

1. INTRODUCTION

The aim of the examination is to indicate the possibility of applying statistics in the education process to students. Systematic application of such analyzes could lead to important conclusions regarding the improvement of the teaching process in schools.

In this paper the analysis of the success of pupils of the sixth grade, within the subject of Technical and Informatics Education was carried out. The random sample was founded on 80 pupils of an elementary school and sample elements by observing the following categories: gender, class and obtained grades to each of four examinations carried out within this subject. The used data are more or less similar and the results of some future tests cannot be accurately predicted.

By monitoring these available data in their entirety, they reveal that the medium results manifest certain pattern and stability that serve as the basis of the mathematical theory of statistics and statistical prediction.

2. CONCEPT AND IMPORTANCE OF STATISTICS

One of the basic problems in mathematical statistics is that on the basis of a sample from a certain population, the distribution of the very character of the population occurs, that is, the theoretical distribution is based on the sample distribution. Of course, information on the theoretical distribution function is obtained with a certain accuracy, or with a certain level of confidence. This accuracy and precision of the conclusions reached can be increased by selecting a sample of sufficiently large volumes. However, we are not always able to get a sample of the desired volume, so our capabilities in terms of accuracy are limited [1].

Since the statistical distribution of a certain feature is precisely determined by the parameters of this distribution, the first problem we face in relation to the observed feature is: an assessment of the unknown parameters of the observed samples.

Basically, our goal is to find the best mark for the parameter, based on the sample. The specification of these values is carried out using statistical hypotheses, and deciding on the acceptability of these values based on the observed sample is done using criteria called statistical tests.

Therefore, if we want to test the hypothesis of a certain value of the mathematical expectation for a particular feature X $(H_0: m = m_0)$, then we first need to evaluate the mathematical expectation for that characteristic X. We use statistics for this assessment

$$\bar{X}_n = \frac{1}{n}\sum_{k=1}^{n} X_k$$

(the medium value of the sample), which represents a centered (unbiased) assessment of the mathematical expectation of that feature X.

In other words, the sample average value serves as the starting point for all tests. Through testing, we only determine how far we can deviate from this average value, and that the starting hypothesis H_0 remains effective. We do this for a certain significance threshold α, usually α = 0.05, i.e. we give certain guarantees for our conclusions.

Our research and the results that follow, fully agree with the reasoned methodology [1], [2].

3. DESCRIPTIVE STATISTICS

Descriptive statistical analysis is a set of methods for calculating, displaying and describing the basic characteristics of statistical series [8], [2].

Descriptive statistical analysis has the following tasks:

- grouping and compiling statistical data,
- displaying statistical data,
- determination of basic indicators of statistical series.

Grouping data is done according to the values or modalities of the observed features. The final result is the statistical series.

The statistical series is a set of variations of the characteristics of the observed statistical mass. The statistical series is presented in the form of a table, in at least two rows and two columns, where the qualitative side of the statistical mass is shown in the first column, and in the second quantitative (numerical) side.

Descriptive statistics include calculating:

- counting measure (frequency and percentage),
- measure of central tendency (mode, median and arithmetic mean) and
- measure of variability (range and standard deviation).

Graphical data representations are called diagrams or charts, and we can divide them into dot, line and surface.

4. METODOLOGY OF RESEARCH

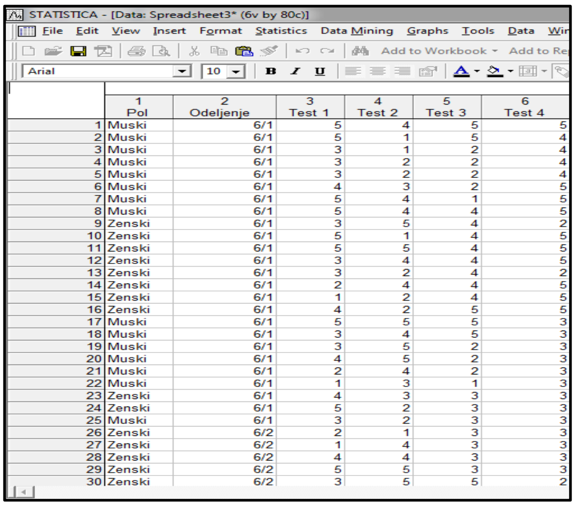
In this paper both descriptive and inferential statistics were used as methods of research. By the method of descriptive statistical analysis, testing of statistical hypotheses, using the Pirson χ^2-test, the contingency table, determining the confidence interval of the expected value, as well as the procedure for creation of a sub-sample, the main results of this paper have been reached.

The data on which the analysis was carried out were taken from the primary school "Sveti Sava" in Čačak, and represent the grades from the subject "Technical and informatics education". The sample consists of 80 pupils on which the six values observed are registered: gender, class (as attribute features) and grades on Test1, Test2, Test3 and Test4 (as numerical features).

The analysis was carried out in the program package *Statistica*, which offers the most comprehensible range of data analysis, data management, data visualization and data extraction procedures. Her arsenal includes the most comprehensible selection of techniques of prognostic modeling, grouping, classification and research methods in the same software platform.

5. APPLICATION OF DESCRIPTIVE STATISTICS IN THE BASIC EDUCATION

This paper analyzes the success of pupils of the sixth grade of "Sveti Sava" elementary school in the field of Technical and informatics education. The sample consists of 80 pupils of this school and for each of these pupils 6 values are registered: gender, class and grades on each of the 4 test. Part of this table is shown in Figure 1 [3], [4], [5], [7]. We can say that pupils' grades are in the maximum interval, or from 1 to 5.

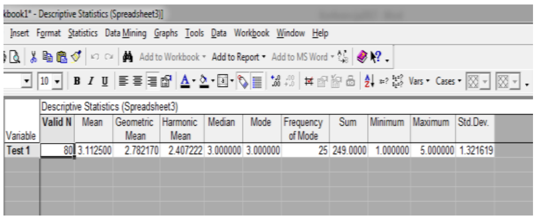


	1 Pol	2 Odeljenje	3 Test 1	4 Test 2	5 Test 3	6 Test 4
1	Muski	6/1	5	4	5	5
2	Muski	6/1	5	1	5	4
3	Muski	6/1	3	1	2	4
4	Muski	6/1	3	2	2	4
5	Muski	6/1	3	2	2	4
6	Muski	6/1	4	3	2	5
7	Muski	6/1	5	4	1	5
8	Muski	6/1	5	4	4	5
9	Zenski	6/1	3	5	4	2
10	Zenski	6/1	5	1	4	5
11	Zenski	6/1	5	5	4	5
12	Zenski	6/1	3	4	4	5
13	Zenski	6/1	3	2	4	2
14	Zenski	6/1	2	4	4	5
15	Zenski	6/1	1	2	4	5
16	Zenski	6/1	4	2	5	5
17	Muski	6/1	5	5	5	3
18	Muski	6/1	3	4	5	3
19	Muski	6/1	3	5	2	3
20	Muski	6/1	4	5	2	3
21	Muski	6/1	2	4	2	3
22	Muski	6/1	1	3	1	3
23	Zenski	6/1	4	3	3	3
24	Zenski	6/1	5	2	3	3
25	Muski	6/1	3	2	3	3
26	Zenski	6/2	2	1	3	3
27	Zenski	6/2	1	4	3	3
28	Zenski	6/2	4	4	3	3
29	Zenski	6/2	5	5	3	3
30	Zenski	6/2	3	5	5	2

Figure 1. *Part of the sample with the values of the observed variables*

As already mentioned, data can be displayed in different ways depending on the tool selection - tabular or graphic (diagram). Summary Statistics provides the possibility of tabular display of the number of individuals in the sample (in this case, the number of pupils in the sixth grade), then medium, minimum and maximum values, as well as the summarized value of the observed sample mark (in this case, the test on the subjects Technical and Informational Education).

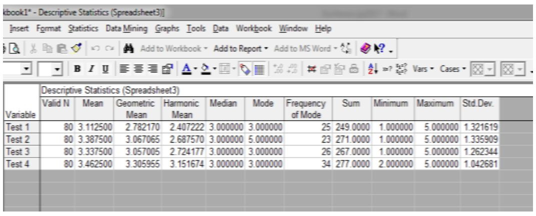
Also, the values of mode, medium and standard deviation are tabulated. In Figure 2, a summary of the *Summary Statistics* for the numerical value of the Test 1 - mark from the first test in the Technical and Informatics course is shown.



Variable	Valid N	Mean	Geometric Mean	Harmonic Mean	Median	Mode	Frequency of Mode	Sum	Minimum	Maximum	Std.Dev.
Test 1	80	3.112500	2.782170	2.407222	3.000000	3.000000	25	249.0000	1.000000	5.000000	1.321619

Figure 2. *Summary Statistics part of numerical variables - Test 1*

In all four tests, the minimum score is 1 (inadequate grade), and the maximum score is 5 (excellent). The average marks for the tests are: Test 1 (3.11), Test 2 (3.38), Test 3 (3.34), Test 4 (3.46) from Technical and Informatics Education (Figure 3).



kbook1* - Descriptive Statistics (Spreadsheet3)]

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Descriptive Statistics (Spreadsheet3)

Variable	Valid N	Mean	Geometric Mean	Harmonic Mean	Median	Mode	Frequency of Mode	Sum	Minimum	Maximum	Std.Dev.
Test 1	80	3.112500	2.782170	2.407222	3.000000	3.000000	25	249.0000	1.000000	5.000000	1.321619
Test 2	80	3.387500	3.067065	2.687570	3.000000	5.000000	23	271.0000	1.000000	5.000000	1.335909
Test 3	80	3.337500	3.057005	2.724177	3.000000	3.000000	26	267.0000	1.000000	5.000000	1.262344
Test 4	80	3.462500	3.305955	3.151674	3.000000	3.000000	34	277.0000	2.000000	5.000000	1.042681

Figure 3. *Summary Statistics for all numerical variables*

The absolute frequency histogram for Test 1 provides a graphical representation of the values of the observed random variables (in this case, Test 1). A histogram is a type of surface diagram showing the values of a random variable. This diagram consists of a column, where the width of the column is the width of the class interval, and the height of the column represents the absolute frequency. By connecting the center of the column, a polygonal line is obtained.

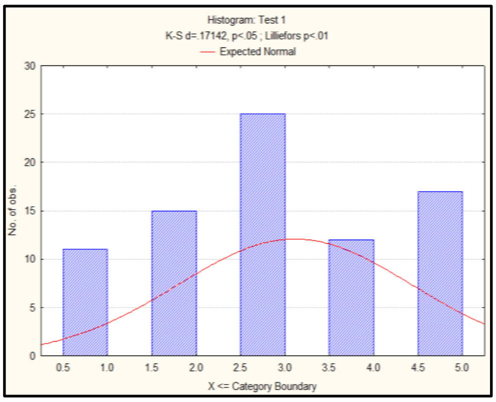


Figure 4. *Absolute frequency histogram for Test 1*

The shape of this polygonal line would change when a new sample of pupils from the same school was formed, but these changes would probably not be statistically significant. The theoretical density function, shown in Figure 4, represents an approximate function for all these polygonal lines. It is obvious that she points to Gauss's (normal) distribution. This assumption is confirmed or denied by the Pirson χ^2-test, the results of which will be presented in the next section.

To mark - a test on Test 1 give another display. This is a Box & Whisker Plot (Figure 5).

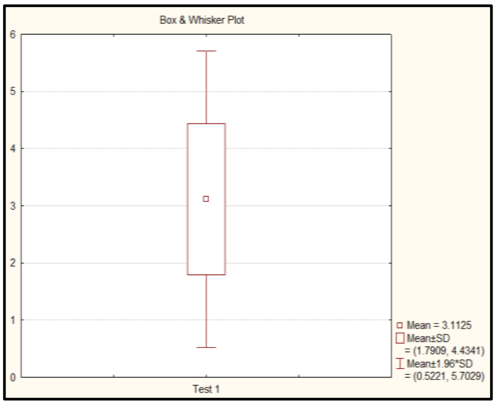


Figure 5. *Box diagram for variables Test 1*

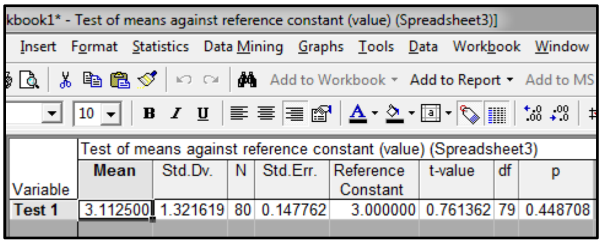
Box & Whisker Plot Diagram (Figure 5) consists of three parts: the middle square - showing the average grade on Test 1 (Mean = 3.1125), box frame - extending to the mean value ± deviation (Mean + SD) and "Whisker" - which reach the mean value of ± 1.96 * standard - deviation (Mean ± 1.96 * SD), which represents the confidence interval for the mathematical expectation of the random variable of this feature.

The small square represents the mean value of the marks obtained in the first test (Mean = 3.1125). The rectangle represents the minimum and maximum deviation from the mean value for the standard deviation value of 1.101710. The ends of the mustache show extreme values that represent values that are outside of the deviation framework. In fact, this is the maximum value of the interval value of this feature.

5.1 Testing statistical hypotheses

5.1.1 Test H_0 ($m = m_0$) against H_1 ($m \neq m_0$) with unknown σ^2

The test relates to a random variable Test 1, which presents the grades that pupils achieved at the first test in the subject Technical and Informatics Education. The H_0 hypothesis is tested (assumption) that the mean or the average grade for this test per student is equal to 3, i.e. $H_0(m_0 = 3)$. The value 3 $(m_0 = 3)$ is entered in the field Test all means against. Then the minimum value of the probability of occurrence of the error of the first type (α minimum) is to be assigned, i.e. the value α = 0.05 (α-level for highlighting) is fixed. After this setting, it is necessary to confirm the Summary button, and then the table shown in Figure 6 is obtained.



kbook1* - Test of means against reference constant (value) (Spreadsheet3)]

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Test of means against reference constant (value) (Spreadsheet3)

Variable	Mean	Std.Dv.	N	Std.Err.	Reference Constant	t-value	df	p
Test 1	3.112500	1.321619	80	0.147762	3.000000	0.761362	79	0.448708

Figure 6. *Values T-test-a for variables Test 1*

Since p = 0.448708> 0.05 we have no reason to reject the zero hypothesis and we can consider that the success of the pupils on this test is characterized by a score of 3.

5.1.2 Testing H_0 ($m_1 = m_2$) against H_1 ($m_1 \neq m_2$) when unknown σ_1^2, σ_2^2

From the Statistics menu, the Basic Statistics / Tables option is selected, and t-test is independent by Variables.

Sampling values of values on Test 1 and Test 2 are different $\bar{x}_1$ =3.1125 and $\bar{x}_2$=3.3875.

The question arises: Is this difference statistically significant, with confidence level of 0.95, or significance threshold α=0.05? This is expressed through a hypothesis, $H_0(m_1 = m_2)$ against an alternative $H_1(m_1 \neq m_2)$.

For this we use the t - test (Figure 7).

T-test for Independent Samples (Spreadsheet3) Note: Variables were treated as independent samples											
Group 1 vs. Group 2	Mean Group 1	Mean Group 2	t-value	df	p	Valid N Group 1	Valid N Group 2	Std.Dev. Group 1	Std.Dev. Group 2	F-ratio Variances	p Variances
Test 1 vs. Test 2	3.112500	3.387500	-1.30891	158	0.192467	80	80	1.321619	1.335909	1.021741	0.924091

Figure 7. *Table showing the results of the test for independent random variables Test 1 and Test 2*

From the table in Figure 7 it can be noticed that the hypothesis H_0 is not accepted, since the value p = 0.192467 (less than 0.05). This test can also be illustrated by a diagram of the boxes, shown in Figure 8, and from which it is also noted that the variables Test1 and Test2 are different. This means that with a confidence of 95% we reject a zero hypothesis $H_0(m_1 = m_2)$ against the alternative $H_1(m_1 \neq m_2)$.

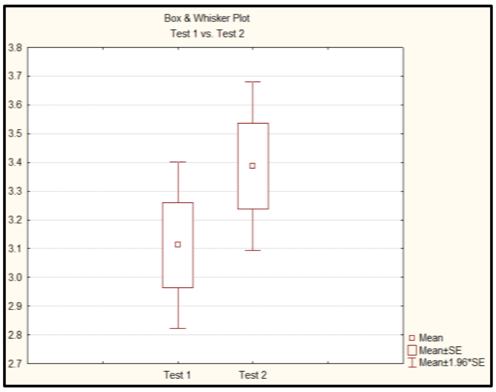

Figure 8. *Box plot for variables Test 1, Test 2*

5.2 Pirson χ^2-test

The Pirson test in the *Statistica* software package shows which types of distribution are supported for a particular pattern [6], [8]. Based on histograms, we can assume the distribution of the observed features. Is this so checked by Pirson's χ^2-test? We test the hypothesis that the data on Test1 should be arranged according to normal distribution.

Variable: Test 1, Distribution: Normal (Spreadsheet3) Chi-Square = 84.20132, df = 6 (adjusted) , p = 0.00000									
Upper Boundary	**Observed Frequency**	Cumulative Observed	Percent Observed	Cumul. % Observed	Expected Frequency	Cumulative Expected	Percent Expected	Cumul. % Expected	Observed-Expected
<= 1.00000	11	11	13.75000	13.7500	4.39800	4.39800	5.49750	5.4975	6.6020
1.50000	0	11	0.00000	13.7500	4.49917	8.89717	5.62396	11.1215	-4.4992
2.00000	15	26	18.75000	32.5000	7.09949	15.99666	8.87437	19.9958	7.9005
2.50000	0	26	0.00000	32.5000	9.72511	25.72177	12.15639	32.1522	-9.7251
3.00000	25	51	31.25000	63.7500	11.56478	37.28655	14.45597	46.6082	13.4352
3.50000	0	51	0.00000	63.7500	11.93871	49.22526	14.92339	61.5316	-11.9387
4.00000	12	63	15.00000	78.7500	10.69929	59.92455	13.37411	74.9057	1.3007
4.50000	0	63	0.00000	78.7500	8.32394	68.24849	10.40493	85.3106	-8.3239
5.00000	17	80	21.25000	100.0000	5.62182	73.87031	7.02727	92.3379	11.3782
< Infinity	0	80	0.00000	100.0000	6.12969	80.00000	7.66211	100.0000	-6.1297

Figure 9.Table of absolute functions Test 1 variables and the results of testing their compliance with the normal distribution

On the basis of the table shown in Figure 9, the absolute frequency histogram is shown in Figure 4 and can be concluded on the basis of the value p of 0, which is less than 0.05 that the data, in this case the estimates, are not distributed by the normal distribution .

Let's examine the possible agreement with Puason's distribution for the same significance threshold α = 0.05.

Variable: Test 1, Distribution: Poisson, Lambda = 3.11 (Spreadsheet3) Chi-Square = 4.31661, df = 3 (adjusted) , p = 0.22924									
Category	Observed Frequency	Cumulative Observed	Percent Observed	Cumul. % Observed	Expected Frequency	Cumulative Expected	Percent Expected	Cumul. % Expected	Observed-Expected
<= 0.00000	0	0	0.00000	0.0000	3.55917	3.55917	4.44896	4.4490	-3.55917
1.00000	11	11	13.75000	13.7500	11.07791	14.63708	13.84739	18.2963	-0.07791
2.00000	15	26	18.75000	32.5000	17.24000	31.87707	21.55000	39.8463	-2.24000
3.00000	25	51	31.25000	63.7500	17.88649	49.76357	22.35812	62.2045	7.11351
4.00000	12	63	15.00000	78.7500	13.91793	63.68150	17.39741	79.6019	-1.91793
< Infinity	17	80	21.25000	100.0000	16.31850	80.00000	20.39813	100.0000	0.68150

Figure 10. *Frequencygroup table for Pauson distribution*

From the table of grouped frequencies in Figure 10 it can be seen that the value p = 0.22924> 0.05. On the basis of this, it can be concluded that the estimates on Test1 were calculated according to Puason's law. The same conclusion applies to Test2, where p = 0.45624> 0.05. However, this does not apply to Test3 and Test4.

Using the χ^2-test of independence, we will test the test results of half of the pupils. First, we will create a contiguity table for the half-marker marks and the results on Test1 (Figure 11).

Summary Frequency Table (Spreadsheet3) Marked cells have counts > 10 (Marginal summaries are not marked)						
Pol	Test 1 1	Test 1 2	Test 1 3	Test 1 4	Test 1 5	Row Totals
Muski	7	8	14	6	7	42
Zenski	4	7	11	6	10	38
All Grps	11	15	25	12	17	80

Statistic	Statistics: Pol(2) x Test 1(5) (Spreadsheet3) Chi-square	df	p
Pearson Chi-square	1.578206	df=4	p=.81270
M-L Chi-square	1.588335	df=4	p=.81089

Figure 11. *Contigency table and independence test*

The results of the test show that Test 1 has a high degree of independence of the test results from half of the pupils because p = 0.81270> 0.05. A similar conclusion applies to Tests 2 and 4, but this independence is less exposed. However, this does not apply to Test 3. This shows which group is more interested in this segment of teaching material, boys or girls. Which gender is exactly the issue depends on who has a higher average grade, boys or girls.

In the end, the independence of the results on the observed tests was investigated by methods of factor analysis, [6], [8] (Figure 12). The results of these analyses have shown that the results in the tests are largely independent (p = 0.27410> 0.05). The only dependence was recorded between the results on Test 1 and Test 4. The reason for this dependence could be subsequently analyzed, but what we need is further information.

Cell No.	Test 2; Unweighted Means (Spreadsheet3) Current effect: F(4, 75)=1.3098, p=.27410 Effective hypothesis decomposition Test 2	Test 1 Mean	Test 1 Std.Err.	Test 1 -95.00%	Test 1 +95.00%	N
1	1	3.142857	0.495652	2.155467	4.130247	7
2	2	2.882353	0.318055	2.248756	3.515950	17
3	3	3.705882	0.318055	3.072285	4.339480	17
4	4	2.750000	0.327843	2.096903	3.403097	16
5	5	3.086957	0.273440	2.542236	3.631677	23

Figure 12. *Factor analysis test table*

6. CONCLUSION

Without computers and statistical programs, processing large amounts of data would be a slow and time-consuming process. Application of statistical packages, such as *Statistica*, enables easier data processing and analysis of results.

The test results mostly do not depend on half of total number of all the pupils. More importantly, the results in the tests are mutually independent. This points to the fact that these results do not depend much on pupil's knowledge on the particular subject, which somehow points to the structure of the subject.

From the results of the analysis we conclude that the results of the test do not depend on half the students. More importantly, the results in the tests are mutually independent.

This points to the fact that these results do not depend much on the knowledge of students on the subject, which somewhat points to the structure of the subject.

For more reliable conclusions, we would need more information and more repeated student testing. Systematic application of such analyzes could lead to important conclusions regarding the improvement of the teaching process in schools.

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